Activity 1 Answers

1. Now $(3n)^2 = 9n^2$ which is obviously divisible by 3

$(3n \pm 1)^2 = 9n^2 \pm 6n + 1 = 3(3n^2 \pm 2) + 1$

2 Any number can be written in the form $5n$, $5n \pm 1$, $5n \pm 2$

Squaring gives $25n^2$, $25n^2 \pm 10n + 1 = 5(5n^2 \pm 2) + 1$, $25n^2 \pm 20n + 4 = 5(5n^2 \pm 4) + 4$

Exhaustion:

N	0	1	2	3	4	5	6	7	8	9
N^2	0	1	4	9	6	5	6	9	4	1
Rem	0	1	4	4	1	0	1	4	4	1

3 $(x - y)(x + y)$

$(x - y)(x + y) = 1 \times p$ so $x - y = 1, \quad x + y = p$

$x = \frac{p+1}{2}, \quad y = \frac{p-1}{2}$

Since *p* is odd, both *x* and *y* are integers.

4

x	$y = \sqrt{24 - x^2}$	
1	$\sqrt{23}$	×
2	$\sqrt{20}$	×
3	$\sqrt{15}$	×
4	$\sqrt{8}$	×
5	×	×

5 1, 2, 4, 8, 16, 32, 64, 128, 256, 512

6 If $n > 0$ then $\frac{dy}{dx} = nx^{n-1} \neq \frac{1}{x}$ because *n* - 1 = -1 implies *n* = 0

If *n* = 0 then $\frac{dy}{dx} = 0$

If *n* < 0 then *n* - 1 < -1

Activity 1 Answers

7

N	0	1	2	3	4	5	6	7	8	9
N^2	0	1	4	9	6	5	6	9	4	1
$2N^2$	0	2	8	8	2	0	2	8	8	2
$5N^2$	0	5	0	5	0	5	0	5	0	5

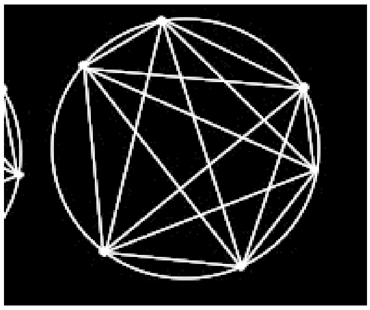
	$2N^2$			
$5N^2$	+	**0**	**2**	**8**
	0	0	2	8
	5	5	7	3

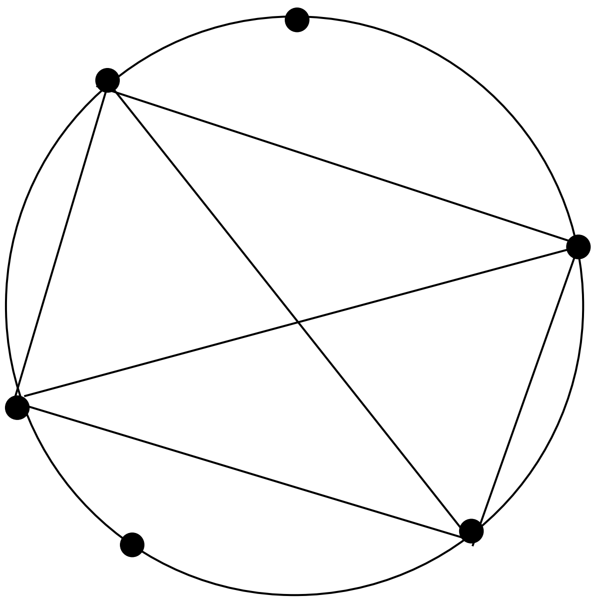
8 As in the example in the presentation $n = 17$ gives $p(17) = 17^2 + 17 + 17 = 17 \times 19$

($n = 16$ gives $p(16) = 16^2 + 16 + 17 = 16(16 + 1) + 17 = 17 \times 17$

9 The numbers get bigger very quickly. F_n denotes the nth *F*ermat number. He believed they were all prime. Euler found that 641 was a factor of F_5

10 You have to look at $n = 6$ and join the dots!





11 The counterexamples are 4 and 8 (and then 16, by exhaustion) .

So the numerical evidence is that the claim is true for all n ∈ ℕ apart from powers of 2

12 (a) $x = 30^{\circ}$ or $x = 45^{\circ}$ are fairly obvious.

(b) $a = 9$, $b = 16$ is a non calculator one.

13 Prove that the equation $2x^2 + kx + k^2 = 0$, $k \neq 0$ has no real roots in x

Using the already established result that the discriminant must be non-negative for real roots

$$k^2 - 4 \times 2 \times k^2 = -7k^2 < 0 \quad \text{so no real roots}$$

Alternatively

$$2x^2 + kx + k^2 = 0 \Rightarrow x^2 + \frac{k}{2}x + \frac{k^2}{2} = 0 \Rightarrow \left(x + \frac{k}{4}\right)^2 + \frac{7k^2}{16} = 0$$ which is impossible as the LHS as the sum of two squares cannot be zero

14 (a) Always true. $y = 0$ when $x = a$ and $y' = 0$ when $x = a$

(b) Only true if $x > 0$

(c) Only true if $-1 < r < 1$